📖 Detailed Solutions (PTS-22)

SECTION A

01. (c) Since $P_{3\times2}A_{m\times n}=Q_{3\times2}$ implies, $2=m$ and $n=2$.
Therefore, the order of A must be 2×2.

02. (b) Since $A(adj.A)=\begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{bmatrix}=-2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=-2I_3$

Also, $A(adj.A)=|A|\,I_3$

$\therefore\ |A|=-2$.

03. (c) As $\vec{a}$ and $\vec{b}$ are collinear so, $\dfrac{\alpha}{2}=\dfrac{3}{-1}=\dfrac{-6}{-\beta}$

Considering $\dfrac{\alpha}{2}=\dfrac{3}{-1},\ \dfrac{3}{-1}=\dfrac{-6}{-\beta}$

$\Rightarrow \alpha=-6,\ \beta=-2$

$\therefore (\alpha+\beta)=-8$.

04. (c) As the greatest integer function is not differentiable at integral points.
So, here $f(x)$ will be non-differentiable at $x=1$.

05. (b) $\int e^x\left(\log\sqrt{x}+\dfrac{1}{2x}\right)dx=e^x\times\log\sqrt{x}+C$.

Using $\int e^x[f(x)+f'(x)]dx=e^xf(x)+C$, where $f(x)=\log\sqrt{x}$, $f'(x)=\dfrac{1}{2x}$.

06. (a) As there are no arbitrary constants in the particular solution of a differential equation so, the number of arbitrary constants is 0.

07. (b) a linear function to be optimized.

08. (c) Required length of perpendicular drawn from $(4,-7,3)$ on y-axis $=\sqrt{4^2+3^2}=5$ units.

09. (c) $\int_1^e\dfrac{\log x}{x}dx=\dfrac{1}{2}\left[(\log x)^2\right]_1^e=\dfrac{1}{2}\left[(\log e)^2-(\log 1)^2\right]$

$=\dfrac{1}{2}[1-0]=\dfrac{1}{2}$.

10. (d) $\begin{vmatrix}2&3&2\\x&x&x\\4&9&1\end{vmatrix}+3=0$

On expanding along R_1, we get : $2(x-9x)-3(x-4x)+2(9x-4x)+3=0$

$\Rightarrow 2(-8x)-3(-3x)+2(5x)+3=0$

$\Rightarrow -16x+9x+10x+3=0$

$\Rightarrow 3x+3=0$

$\therefore x=-1$.

11. (a) As maximum value of z occurs at (2, 4) and (4, 0) so, $a(2)+b(4)=a(4)+b(0)$

$\Rightarrow a=2b$.

12. (c) As $\begin{bmatrix}2&-1&3\\\lambda&0&7\\-1&1&4\end{bmatrix}$ is non-invertible so, $\begin{vmatrix}2&-1&3\\\lambda&0&7\\-1&1&4\end{vmatrix}=0$

$\Rightarrow 2(-7)+1(4\lambda+7)+3(\lambda)=0 \qquad \therefore\ \lambda=1$.

13. (a) $a_{12} + a_{22} = |1^2 - 2| + |2^2 - 2| = 1 + 2 = 3$.

14. (b) Re-writing the D.E., $\dfrac{dy}{dx} + \dfrac{y}{\sqrt{x}} = \dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}$

On comparing to $\dfrac{dy}{dx} + P\,y = Q$, we observe $P = \dfrac{1}{\sqrt{x}}$ and $Q = \dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$\therefore$ Integration factor $= e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$.

15. (c) $P(B'|A) = \dfrac{P(B' \cap A)}{P(A)}$

$\Rightarrow P(B'|A) = \dfrac{P(A) - P(A \cap B)}{P(A)}$ $(\because B' \cap A = A - B$

$\Rightarrow P(B'|A) = \dfrac{P(A) - P(A) \times P(B)}{P(A)}$ ($\because$ A and B are independent events

$\Rightarrow P(B'|A) = 1 - P(B)$

$\Rightarrow P(B'|A) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$.

16. (c) f (x) is discontinuous at exactly three points, $x = 0, 1, -1$.

17. (d) As the number of Reflexive relations defined on a set of n elements $= 2^{n(n-1)}$.
So, 2^6 reflexive relations are possible in the set A where $n(A) = 3$.

18. (b) Equation of line joining $(-1, 3, 2)$ and $(5, 0, 6)$ is $\dfrac{x+1}{6} = \dfrac{y-3}{-3} = \dfrac{z-2}{4} = \lambda$

The random point on the line is $(6\lambda - 1, -3\lambda + 3, 4\lambda + 2)$.

As x-coordinate of point P is 2 so, $6\lambda - 1 = 2 \Rightarrow \lambda = \dfrac{1}{2}$

Therefore, the z-coordinate is $4\lambda + 2 = 4\left(\dfrac{1}{2}\right) + 2 = 4$.

19. (b) Here both A and R are true and R is not the correct explanation of A.

20. (c) $\overrightarrow{OP} = \dfrac{2\overrightarrow{OB} + 1\overrightarrow{OA}}{2+1}$

$\Rightarrow \overrightarrow{OP} = \dfrac{2(2\hat{i} - \hat{j} + 2\hat{k}) + 1(2\hat{i} - \hat{j} - \hat{k})}{3}$

$\therefore$ $\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k}$. So, A is true.

Also, R is false. Since $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

SECTION B

21. $\sin^{-1}\left[\sin\left(-\dfrac{17\pi}{8}\right)\right] = \sin^{-1}\left[\sin\left(-2\pi - \dfrac{\pi}{8}\right)\right] = \sin^{-1}\left[-\sin\left(\dfrac{\pi}{8}\right)\right] = -\sin^{-1}\left(\sin\dfrac{\pi}{8}\right) = -\dfrac{\pi}{8}$.

OR

Note that the domain of $\sin^{-1} x$ is $x \in [-1, 1]$ and that of $\tan^{-1} x$ is $x \in R$.

So, the domain of $f(x) = \tan^{-1} x + \dfrac{1}{2}\sin^{-1} x$ is $x \in [-1, 1]$.

Also, note that $\sin^{-1}x$ and $\tan^{-1}x$ both are increasing functions in the interval $x \in [-1, 1]$.
So, at $x = -1$, we get the minimum value of $f(x)$ i.e.,

$$f(-1) = \tan^{-1}(-1) + \frac{1}{2}\sin^{-1}(-1) = -\frac{\pi}{4} + \frac{1}{2}\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2}$$

and, at $x = 1$, we get the maximum value of $f(x)$ i.e.,

$$f(1) = \tan^{-1}(1) + \frac{1}{2}\sin^{-1}(1) = \frac{\pi}{4} + \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{2}.$$

Hence, the range of $f(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

22. Let a denote the side length of the triangle so, $\frac{da}{dt} = 2$ cm/s.

Since area of the equilateral triangle is $A = \frac{\sqrt{3}\,a^2}{4}$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}\,a}{2} \times \frac{da}{dt}$$

$$\therefore \left.\frac{dA}{dt}\right]_{\text{at } a=20\text{ cm}} = \frac{\sqrt{3}\times 20}{2} \times 2 = 20\sqrt{3}\ \text{cm}^2/\text{s}.$$

23. $\left\{|\vec{a}|\,\vec{b} + |\vec{b}|\,\vec{a}\right\}.\left\{|\vec{a}|\,\vec{b} - |\vec{b}|\,\vec{a}\right\}$

$\Rightarrow \quad = |\vec{a}|^2\,\vec{b}.\vec{b} - |\vec{a}||\vec{b}|\,\vec{b}.\vec{a} + |\vec{b}||\vec{a}|\,\vec{a}.\vec{b} - |\vec{b}|^2\,\vec{a}.\vec{a}$

$\Rightarrow \quad = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}||\vec{b}|\vec{a}.\vec{b} + |\vec{a}||\vec{b}|\vec{a}.\vec{b} - |\vec{b}|^2|\vec{a}|^2$

$\Rightarrow \quad = 0$.

Hence $\left\{|\vec{a}|\,\vec{b} + |\vec{b}|\,\vec{a}\right\} \perp \left\{|\vec{a}|\,\vec{b} - |\vec{b}|\,\vec{a}\right\}$ as, $\vec{a}$ and $\vec{b}$ are non-zero vectors.

OR

The vector equation of the line is $\vec{r} = 3\hat{i} + 4\hat{j} + 5\hat{k} + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$.

\# Note we have used $\vec{r} = \vec{a} + \lambda\vec{b}$.

Also, Cartesian equation of the line is $\frac{x-3}{2} = \frac{y-4}{2} = \frac{z-5}{-3}$.

24. $y = e^x + e^{-x}$

On squaring both sides, $y^2 = e^{2x} + e^{-2x} + 2e^x e^{-x}$ i.e., $y^2 = e^{2x} + e^{-2x} + 2$

$\Rightarrow y^2 - 4 = e^{2x} + e^{-2x} - 2$

$\Rightarrow y^2 - 4 = (e^x - e^{-x})^2 \ldots(i)$

Now $y = e^x + e^{-x}$ implies, $\frac{dy}{dx} = e^x - e^{-x}$

By (i), we get : $\frac{dy}{dx} = \sqrt{y^2 - 4}$.

25. $$\frac{\text{Projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{Projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{\vec{a}.\hat{b}}{\vec{b}.\hat{a}} = \frac{\vec{a}.\frac{\vec{b}}{|\vec{b}|}}{\vec{b}.\frac{\vec{a}}{|\vec{a}|}} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} \times \frac{|\vec{a}|}{\vec{b}.\vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$$

$$\Rightarrow \quad = \frac{\sqrt{4+1+4}}{\sqrt{25+9+16}} = \frac{3}{5\sqrt{2}}.$$

So, the ratio is $3:5\sqrt{2}$.

SECTION C

26. Let $I = \int \frac{x^3+1}{x^3-x}\,dx$

$$\Rightarrow I = \int \frac{x^3-x+x+1}{x^3-x}\,dx$$

$$\Rightarrow I = \int \left(\frac{x^3-x}{x^3-x} + \frac{x+1}{x^3-x}\right) dx$$

$$\Rightarrow I = \int \left(1 + \frac{1}{x(x-1)}\right) dx$$

$$\Rightarrow I = \int \left(1 + \frac{1}{x-1} - \frac{1}{x}\right) dx$$

$$\Rightarrow I = x + \log|x-1| - \log|x| + C \text{ or, } x + \log\left|\frac{x-1}{x}\right| + C.$$

27. Let E_1 : the lost card is king, E_2 : the lost card is not a king and, A : two cards drawn are kings.

Here $P(E_1) = \frac{4}{52} = \frac{1}{13}$, $P(E_2) = 1 - \frac{1}{13} = \frac{12}{13}$; $P(A|E_1) = \frac{{}^3C_2}{{}^{51}C_2}$, $P(A|E_2) = \frac{{}^4C_2}{{}^{51}C_2}$

By Bayes' theorem, $P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)}$

$$\Rightarrow P(E_1|A) = \frac{\frac{1}{13} \times \frac{3}{51\times 25}}{\frac{1}{13} \times \frac{3}{51\times 25} + \frac{12}{13} \times \frac{6}{51\times 25}}$$

$$\Rightarrow P(E_1|A) = \frac{3}{75}$$

$$\therefore P(E_1|A) = \frac{1}{25}.$$

OR

Let $X =$ Number of white balls.
So, values of $X = 0, 1, 2$.
Table for probability distribution is :

X	0	1	2
P(X)	$\frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}=\frac{7}{15}$	$3\times\frac{8}{10}\times\frac{7}{9}\times\frac{2}{8}=\frac{7}{15}$	$3\times\frac{2}{10}\times\frac{1}{9}\times\frac{8}{8}=\frac{1}{15}$

Now mean $= \sum X.P(X) = 0\times\frac{7}{15} + 1\times\frac{7}{15} + 2\times\frac{1}{15} = \frac{9}{15} = \frac{3}{5}$.

28. Let $f(x) = |x| + |x+1| + |x-5|$

$$\Rightarrow f(x) = \begin{cases} -x+x+1-(x-5) = 6-x, \text{ if } -1 \le x \le 0 \\ x+x+1-(x-5) = 6+x, \text{ if } 0 \le x \le 5 \end{cases}$$

Now $\int_{-1}^{5}\left(|x|+|x+1|+|x-5|\right)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{5} f(x)dx$

$\Rightarrow \quad = \int_{-1}^{0}(6-x)dx + \int_{0}^{5}(6+x)dx$

$\Rightarrow \quad = \frac{1}{-2}\left[(6-x)^2\right]_{-1}^{0} + \frac{1}{2}\left[(6+x)^2\right]_{0}^{5}$

$\Rightarrow \quad = -\frac{1}{2}[36-49] + \frac{1}{2}[121-36]$

$\Rightarrow \quad = \frac{13}{2} + \frac{85}{2}$

$\Rightarrow \quad = 49$.

OR

Note that $\tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right) = \tan^{-1}\left(\frac{(1-x)-x}{1+(1-x)x}\right) = \tan^{-1}(1-x) - \tan^{-1}x$

Let $I = \int_{0}^{1} \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx$

$\Rightarrow I = \int_{0}^{1}\left[\tan^{-1}(1-x) - \tan^{-1}x\right]dx$

$\Rightarrow I = \int_{0}^{1}\tan^{-1}[1-(1-x)]dx - \int_{0}^{1}\tan^{-1}x\,dx$ (Using $\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx$ in first integral

$\Rightarrow I = \int_{0}^{1}\tan^{-1}x\,dx - \int_{0}^{1}\tan^{-1}x\,dx$

$\therefore I = 0$.

29. Re-writing the D.E., $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x(vx)}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{-v^2-1}{2v}$

$\Rightarrow \int\frac{2v}{v^2+1}dv = -\int\frac{dx}{x}$

$\Rightarrow \log\left|v^2+1\right| = -\log x + \log C$ $(\because x > 0$

$\Rightarrow \log\left|\frac{y^2}{x^2}+1\right| = -\log x + \log C$

$\Rightarrow \log\left|y^2+x^2\right| - 2\log x = -\log x + \log C$

$\Rightarrow \log\left|y^2+x^2\right| = \log x + \log C$

$\Rightarrow \log\left|y^2+x^2\right| = \log Cx$

$\Rightarrow y^2+x^2 = Cx$.

OR

$\cos y\,dx + (1+e^{-x})\sin y\,dy = 0$

$\Rightarrow \int \frac{dx}{(1+e^{-x})} + \int \tan y dy = 0$

$\Rightarrow \int \frac{e^x\,dx}{(e^x+1)} + \int \tan y dy = 0$

$\Rightarrow \log\left|e^x+1\right| + \log\left|\sec y\right| = \log C$ $\left[\because \int \frac{f'(x)dx}{f(x)} = \log\left|f(x)\right| + C\right.$

$\Rightarrow \log\left|(e^x+1)\sec y\right| = \log C \quad \Rightarrow \left|(e^x+1)\sec y\right| = C$

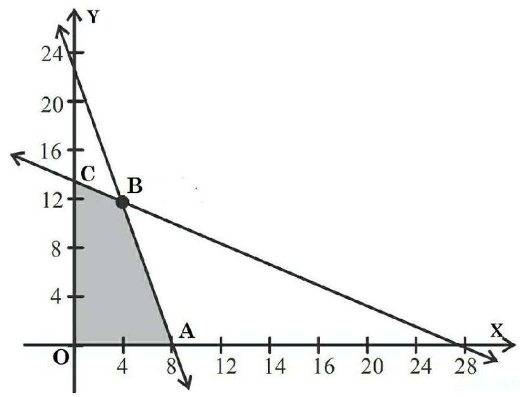
As it is given that $y = \frac{\pi}{4}$ when $x = 0$ so, we have

$\left|(e^0+1)\sec\frac{\pi}{4}\right| = C \quad \Rightarrow C = 2\sqrt{2}$

Therefore, the required solution is given as $\left|(e^x+1)\sec y\right| = 2\sqrt{2}$.

30. Consider the graph shown here.

Corner Points	Value of Z
O(0, 0)	0
A(8, 0)	240
B(4, 12)	360 ← Max.
C(0, 14)	280



Maximum value of Z is 360.

31. $\int \frac{2x+1}{\sqrt{3+2x-x^2}}dx = -\int \frac{2-2x}{\sqrt{3+2x-x^2}}dx + 3\int \frac{1}{\sqrt{2^2-(x-1)^2}}dx$

In first integral, put $3+2x-x^2 = t \Rightarrow (2-2x)dx = dt$

That is, $\int \frac{2x+1}{\sqrt{3+2x-x^2}}dx = -\int \frac{1}{\sqrt{t}}dt + 3\int \frac{1}{\sqrt{2^2-(x-1)^2}}dx$

$= -2\sqrt{t} + 3\sin^{-1}\left(\frac{x-1}{2}\right) + C$

$= -2\sqrt{3+2x-x^2} + 3\sin^{-1}\left(\frac{x-1}{2}\right) + C$.

SECTION D

32. Given that, $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$\Rightarrow |A| = 1(2-2) - 2(3+4) - 3(-3-4) = -14 + 21 = 7$$

Consider A_{ij} as the cofactor of a_{ij}.

$A_{11} = 0,\ A_{12} = -7,\ A_{13} = -7$

$A_{21} = 1,\ A_{22} = 7,\ A_{23} = 5$

$A_{31} = 2,\ A_{32} = -7,\ A_{33} = -4$

$$\therefore \text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \qquad \Rightarrow A^{-1} = \frac{1}{|A|} \times \text{adj}(A) = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The system of equations in Matrix form can be written as

$$A \cdot X = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

By equality of matrices, we get : $x = 1, y = -5, z = -5$.

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow AB = 6I$$

$$\Rightarrow A\left(\frac{1}{6}B\right) = I$$

$$\Rightarrow A^{-1} = \frac{1}{6}(B)$$

The given system of equations is

$x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

That is $AX = D$, where $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow X = A^{-1}D$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

By equality of matrices, we get : $x = 2,\ y = -1,\ z = 4$.

33. Re-writing the given lines, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1},\ \frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$.

Clearly, $\vec{a}_1 = \hat{i} - \hat{j},\ \vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k},\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k},\ \vec{b}_2 = 5\hat{i} + \hat{j}$.

Now $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k},\ \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = -\hat{i} + 5\hat{j} - 13\hat{k}$

$$\therefore\ \text{S.D.} = \frac{\left|(\vec{a}_2 - \vec{a}_1).\vec{b}_1 \times \vec{b}_2\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$\Rightarrow \text{S.D.} = \frac{\left|(-2\hat{i} + 3\hat{j} + 2\hat{k}).(-\hat{i} + 5\hat{j} - 13\hat{k})\right|}{\sqrt{1 + 25 + 169}}$$

$$\Rightarrow \text{S.D.} = \frac{|2 + 15 - 26|}{\sqrt{195}}$$

$$\therefore \text{S.D.} = \frac{9}{\sqrt{195}} \text{ units .}$$

Since $\text{S.D.} \neq 0$ so, the lines do not intersect each other.

OR

Let $L_1 : \frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1} = \lambda$ and $L_2 : \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2} = \mu$.

The coordinates of random points on these lines are given as, $A(\lambda + 2,\ 3\lambda + 2,\ \lambda + 3)$ and $B(\mu + 2,\ 4\mu + 3,\ 2\mu + 4)$ respectively.

If lines intersect then A and B must coincide that means,

$\lambda + 2 = \mu + 2,\ 3\lambda + 2 = 4\mu + 3,\ \lambda + 3 = 2\mu + 4$

$\Rightarrow \lambda = \mu$ ...(i)

$3\lambda - 4\mu = 1$ ...(ii)

$\lambda - 2\mu = 1$ ...(iii)

Solving (i) and (ii), we get : $\lambda = -1,\ \mu = -1$.

Putting $\lambda = -1,\ \mu = -1$ in LHS of (iii) : $\lambda - 2\mu = -1 - 2(-1) = 1 = \text{RHS of (iii)}$.

Hence, A and B will coincide and therefore, the lines L_1 and L_2 will intersect.

Also, the point of intersection will be (1, –1, 2).

34. We have $f(x)=\sin x-\cos x,\ 0<x<2\pi$

$\Rightarrow f'(x)=\cos x+\sin x\ ,\ f''(x)=-\sin x+\cos x$

For local points of maxima and minima $f'(x)=\cos x+\sin x=0$

$\Rightarrow \tan x=-1$

$\therefore x=\frac{3\pi}{4},\frac{7\pi}{4}\in(0,\ 2\pi)$

$\because f''\left(\frac{3\pi}{4}\right)=\cos\frac{3\pi}{4}-\sin\frac{3\pi}{4}=-\sqrt{2}<0$ and, $f''\left(\frac{7\pi}{4}\right)=\cos\frac{7\pi}{4}-\sin\frac{7\pi}{4}=\sqrt{2}>0$

$\therefore f(x)$ is maximum at $x=\frac{3\pi}{4}$ and, minimum at $x=\frac{7\pi}{4}$

So, local maximum value $f\left(\frac{3\pi}{4}\right)=\sin\frac{3\pi}{4}-\cos\frac{3\pi}{4}=\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)=\sqrt{2}$

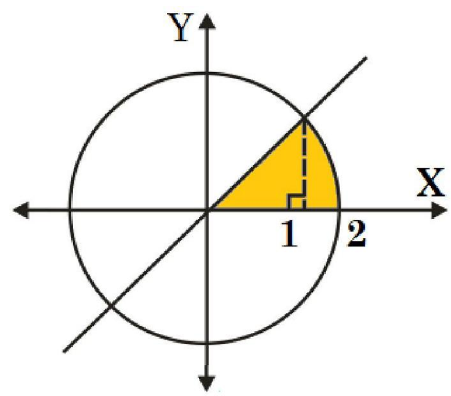
And, local minimum value $f\left(\frac{7\pi}{4}\right)=\sin\frac{7\pi}{4}-\cos\frac{7\pi}{4}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\sqrt{2}$.

35. Solving $y=\sqrt{3}x$ and $x^2+y^2=4$

We get $x^2+3x^2=4$

$\Rightarrow x^2=1$

$\Rightarrow x=1$ (in first quadrant, $x>0$)



Required Area $=\sqrt{3}\int_0^1 x\,dx+\int_1^2\sqrt{2^2-x^2}\,dx$

$=\frac{\sqrt{3}}{2}[x^2]_0^1+\left[\frac{x}{2}\sqrt{2^2-x^2}+2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$

$=\frac{\sqrt{3}}{2}+\left[2\times\frac{\pi}{2}-\frac{\sqrt{3}}{2}-2\times\frac{\pi}{6}\right]$

$=\frac{2\pi}{3}$ Sq. units.

SECTION E

36. **(i)** As X and Y both have exercised their voting right and $X, Y\in I$.

So, $(X,Y)\in R$ is true.

(ii) Since Mr. 'H' and his wife 'W' both have exercised their voting right in the general election.

So, we must have $(H,W)\in R$ and $(W,H)\in R$ always.

Therefore, the given statement is not true.

(iii) The relation R may or may not be a reflexive relation depending upon the factor whether all the citizens belonging to set I have exercised their voting right or not.

For example, if Piyush $\in I$ did not vote then, we will not have (Piyush, Piyush) $\in R$ even if he has voting right (as he belongs to set I).

So, it is not necessary always for R to be a reflexive relation.

Note that, if $(V_1,\ V_2)\in R$ then it means both V_1 and V_2 have exercised their voting right.

That is, if $(V_1,\ V_2)\in R$ then, we will surely have $(V_2,\ V_1)\in R\ \forall V_1,\ V_2\in I$.

Therefore, R is symmetric.

OR

(iii) Note that Mr. Radheshyam did not exercise his voting right although he is eligible to vote.
That means, both Mr. Ghanshyam and Mr. Radheshyam $\in I$.
So, we can not have $(\text{Ghanshyam, Radheshyam}) \in R$. As R is a relation of those citizens who exercised their voting right in general election.
Also, since both Mr. Radheshyam and Miss. Radhika did not vote in the general election so, $(\text{Radhika, Radheshyam}) \notin R$ is a correct statement.

37. **(i)** As $\sum P(x) = 1$ so, $P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+... = 1$
$\Rightarrow 0.2 + k\times1 + k\times2 + k(6-3) + k(6-4) + 0 + ... = 1$
$\Rightarrow 8k = 1 - 0.2$
$\therefore k = 0.1$.
(ii) $P(x \le 1) = P(0)+P(1) = 0.2 + (0.1)(1) = 0.3$.
(iii) $P(x \ge 3) = P(3)+P(4)+P(5)+... = (0.1)(3)+(0.1)(2)+0+... = 0.5$.
Also, $P(x = 2) = (0.1)(2) = 0.2$.

OR

(iii) $P(x \ge 1) = P(1)+P(2)+P(3)+P(4)+P(5)+...$
$\therefore$ Required probability $= (0.1)(1)+(0.1)(2)+(0.1)(3)+(0.1)(2)+0+... = 0.8$.

38. **(i)** As the company increases ₹ x/- in the annual subscription.
$\therefore$ Total revenue generated by company, $R(x) =$ ₹ $(300+x)(500-x)$
$\Rightarrow R(x) = 150000 + 200x - x^2$
$\therefore \frac{d}{dx}\left[R(x)\right] = 200 - 2x$
That is, $R'(x) = 200 - 2x$.
(ii) $\because R'(x) = 200 - 2x$
$\therefore R''(x) = -2$
For $R'(x) = 0,\ 200 - 2x = 0 \quad \therefore x = 100$
Also note that, $R''(100) = -2 < 0$ so, R (x) is maximum at $x = 100$.
Hence, maximum earning will be possible when there is an increment of ₹ 100/-.